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THE IMPORTANCE OF STRUCTURAL AND PRACTICAL UNIDENTIFIABILITY IN MODELING AND TESTING OF AGRICULTURAL MACHINERY. IDENTIFIABILITY TESTING OF AGGREGATE MODEL PARAMETERS TRACTOR BALER-WRAPPER

Summary

New methods of parametric identification are presented in particular tests of identifiability and non-identifiability of model parameters. A definition of the concept of identifiability of model parameters is presented. Methods for testing identifiability using Laplace transform using similarity transformation and using symbolic calculations are described. Available software for testing model identifiability is presented. These are programs for symbolic calculations (MAPLE MATHEMATICA) operating in the form of web applications and in the form of tools for the Matlab environment. The method of introducing the model to the computational environment in the form ordinary differential equations (ODE) is presented. Examples of calculations identifiability of parameters of the complex model of the tractor-single-axle agricultural machine e.g. a baler-wrapper are included.
Keywords*: dynamic modelling, identifiability, parameter identification, agricultural machine*

ZNACZENIE NIEIDENTYFIKOWALNOŚCI STRUKTURALNEJ I PRAKTYCZNEJ W MODELOWANIU I BADANIACH MASZYN ROLNICZYCH. BADANIE IDENTYFIKOWALNOŚCI PARAMETRÓW MODELU AGREGATU CIĄGNIK-PRASOOWIJARKA

Streszczenie

Przedstawiono nowe metody prowadzenia identyfikacji parametrycznej w szczególności badania identyfikowalności oraz nieidentyfikowalności parametrów modelu. Przedstawiono definicję pojęcia identyfikowalności parametrów modelu. Opisano metody badania identyfikowalności za pomocą transformacji Laplace'a z zastosowaniem transformacji podobieństwa oraz za pomocą obliczeń symbolicznych. Przedstawiono dostępne oprogramowanie do badania identyfikowalności modelu. Są to programy do obliczeń symbolicznych (MAPLE MATHEMATICA) działające w formie aplikacji sieciowych oraz w postaci przyborników do środowiska Matlab. Przedstawiono sposób wprowadzenia modelu do środowiska obliczeniowego w postaci równań różniczkowych zwyczajnych. Zamieszczono przykładowe wyniki obliczeń identyfikowalności parametrów złożonego modelu układu ciągnik-jednoosiowa maszyna rolnicza, np. prasoowijarka.
Słowa kluczowe*: modelowanie dynamiczne, identyfikowalność, identyfikacja parametrów, maszyna rolnicza*

1. Introduction

Mathematical modeling is tool used to better understand complex mechanical system. A common problem that arises when developing a model of a mechanical system is that some of its parameters are unknown. This is especially important when those parameters have special meaning but cannot be directly measured. Thus a natural question arises: Can all or at least some of the model's parameters be estimated indirectly and uniquely from observations of the system's input and output? A concept structural identifiability which plays a central role in identification problems was introduced for the first time in the work of Bellman and Åström [1]. The concept is useful when answering questions such as: To what extent is it possible to get insight into the internal structure of a system from input-output measurements? What experiments are necessary in order to determine the internal couplings uniquely? Sometimes the uniqueness holds only within a certain range. In this case we say that a system is only locally structurally identifiable.

2 The concept of model identifiability

The definition of structural identifiability was formulated by Walter and Pronzato (1997) and Ljung [7]. Let us assume that the notation $M_i(p_i) \equiv M_j(p_j)$ means that the model with the structure $M_i(\cdot)$and the values of parameters p_i behaves the same (i.e. for the same input values the same output values are obtained) as the model with the structure $M_j(\cdot)$ and p_j parameter values for any input values and any time t.

$$M(p) = M(p^*) \Rightarrow p = p^* \tag{1}$$

We say that the $M(\cdot)$ model is uniquely identifiable (or globally identifiable) if there is a condition for almost every p^* value.

This means that the identical input / output behavior of two identical model structures implies that the estimated set of parameters is unique and corresponds to the real set of parameters.

Structural identifiability regards the possibility of giving unique values to model unknown parameters from the available observables assuming perfect experimental data (i.e. noise-free and continuous in time).

A parameter p_i i=1,...,n is structurally globally (or uniquely) identifiable if for almost any $p^* \in P$ [3],

$$\sum(p) = \sum(p^*) \Rightarrow p_i = p_i^* \quad . \qquad (1)$$

A parameter p_i i=1,...,n is structurally locally identifiable if for almost any $p^* \in P$ there exists a neighbourhood $V(p^*)$ such that

$$p \in V(p^*) \text{ and } \sum(p) = \sum(p^*) \Rightarrow p_i = p_i^* \quad . \qquad (3)$$

A parameter p_i i=1,...,n is structurally non-identifiable if for almost any $p^* \in P$ there exists no neighbourhood $V(p^*)$ such that

$$p \in V(p^*) \text{ and } \sum(p) = \sum(p^*) \Rightarrow p_i = p_i^* \quad . \qquad (2)$$

Identifiability testing allows the design of experiments providing guidelines for the selection of measurement locations of the inputs and outputs of the system to enable its uniquely identification. This is especially useful for complex systems (e.g. physiological systems) where the number and location of possible inputs and outputs is often very limited.

The Identifiability results can be used to formulate a minimum i.e. necessary and sufficient input / output configuration for complex experiments. Identifiability analysis can also be helpful in providing guidelines for dealing with system identifiability providing guidance on how to simplify model structure or get more information (increasing measured parameters) needed for a particular experiment.

Identifiability analysis is a critical step in the process of parameter estimation it allows you to assess whether it is possible to obtain unique model parameter values from the given set of data.

3. Methods for testing structural identifiability

Structural identifiability analysis of linear models is well understood and there are a number of methods to perform such a task. We will present several methods of testing identifiability based on the analysis of the system described by equations in the state space.

3.1. Identifiability testing using Laplace transforms

The state-space model is represented by a system of equations of the form:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \qquad (5)$$

where: x(t)∈ is a vector of the states of the system u(t) is the vector of inputs y(t) is the vector of outputs.

The A matrix is called the dynamics matrix the B matrix is called the control matrix the C matrix is called the sensor matrix and the D matrix is called the direct term. The one-to-many relationship between a system's transfer function and its state-space model is given by Laplace variable:

$$G(s) = C(sI-A)^{-1}B+D \qquad (6)$$

The Laplace variable state-space model (A,B,C,D) is a minimal realization of G(s) meaning that it is both controllable and observable.

The parameterization $P(\alpha)$ of system matrices (A,B,C,D) is globally identifiable from the transfer function G(s) if for all α_1,α_2 there is a relationship [12]:

$$G(s)=C(\alpha_1)(sI-A(\alpha_1))^{-1}B(\alpha_1)=C(\alpha_2)(sI-A(\alpha_2))^{-1}B(\alpha_2) \qquad (7)$$

then $\alpha_1=\alpha_2$.

For the linear model the classic approach to testing its identifiability is to analyze the transmittance function obtained from the Laplace transform. The matrix of transmittance function is defined as:

$$H(s,p) = \frac{Y(s,p)}{U(s,p)} \quad . \qquad (8)$$

where:
s - is an argument from Laplace space,
Y (s p) U (s p) - are Laplace transformations of output and input.

After writing H (s p) in canonical form we can describe the transmittance as functions of the parameters p and p *. Based on the relation H (s p) = H (s p *) we can derive a set of equations describing the transmittance coefficients H (s p) and H (s p *). If the solution of the system of equations is unique for p then p = p * and the model is structurally identifiable.

As an example of using this method we can analyze a system with one input and one output (SISO) described by the relationships:

$$\frac{dx(t)}{dt} = (a+b)x + cu \quad . \qquad (9)$$

$$x_0 = 0 \text{ and } y(t) = x(t). \qquad (10)$$

The parameters are a b c and input u. The output of y (t) is the state variable x (t). After applying the Laplace transform we get:

$$sX(s) = (a+b)X(s) + cU(s) \quad , \qquad (11)$$

where X (s) U (s) are the state variable and the input variable in the Laplace domain. The observable model in the Laplace domain is Y (s) = X (s). The transmittance function has the form:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{c}{s-(a+b)} \quad . \qquad (12)$$

Knowing the form of transmittance you can identify any quantities that are described by one factor. All quantities that consist of a combination of two or more coefficients are not identifiable by themselves. It is required to specify a set of solutions for quantities composed of several factors. Identification equations take the form:

$$c = c^* \qquad (13)$$

$$a + b = a^* + b^* \quad . \qquad (14)$$

Based on equations (13) and (14) it can be concluded that the parameter c is uniquely identifiable while the parameters a and b are unidentifiable. Equation (14) has an infinite number of solutions.

3.2. System identifiability study using similarity transformation

Walter and Lecourtier proposed the similarity transformation method for linear ODE models. The system is described in state space form:

$$\dot{x} = Ax + Bu \tag{15}$$

$$y = Cx \tag{16}$$

where A B and C are matrices of constant coefficients.

The basic idea of this method is to find the similarity transformation matrix S = P^{-1} AP of matrix A satisfying the equation [10, 14]:

$$\dot{x} = (P^{-1}AP)x + Bu, \tag{17}$$

where P is a non-singular matrix.

It is straight forward to show that if the only possible similar transformation of A is P = I the system is uniquely and globally identifiable; if a finite number of P ≠ I can be found the system is locally identifiable (or nonuniquely identifiable); otherwise the system is unidentifiable.
Before you can apply the similarity transformation method you need to know the controllability and observability of the system. Differential equations should be generated and solved to determine the identifiability of the system. The similarity transformation method is not applicable to general nonlinear systems.

4. Computer systems for testing the identifiability of model parameters

System identifiability testing is also carried out using symbolic calculations. Symbolic calculations can be performed using dedicated computer systems e.g. MAPLE Maxima or MATHEMATICA. In addition to symbolic calculation programs other programs may be used to investigate identifiability.

COMBOS (Fig. 1) is a web app that addresses and solves the structural identifiability problem for a practical class of nonlinear (and linear) ordinary differential equation (ODE). COMBOS use the computer algebra system Maxima and symbolic differential algebra algorithm based on computing Gröbner bases of model attributes.

Matlab toolboxes are also available that use symbolic calculations performed in the MATLAB environment. The best known Matlab toolboxes for identifiability calculations are [2, 8, 13, 15, 16]:
• PottersWheel using the Profile Likelihood Approach method (method of estimating credibility),
• DAISY (Differential Algebra for Identifiability of SYstems),
• GenSSI (Generating Series approach for testing Structural Identifiability),
• STRIKE-GOLDD (STRuctural Identifiability taKen as Extended-Generalized Observability with Lie Derivatives and Decomposition).

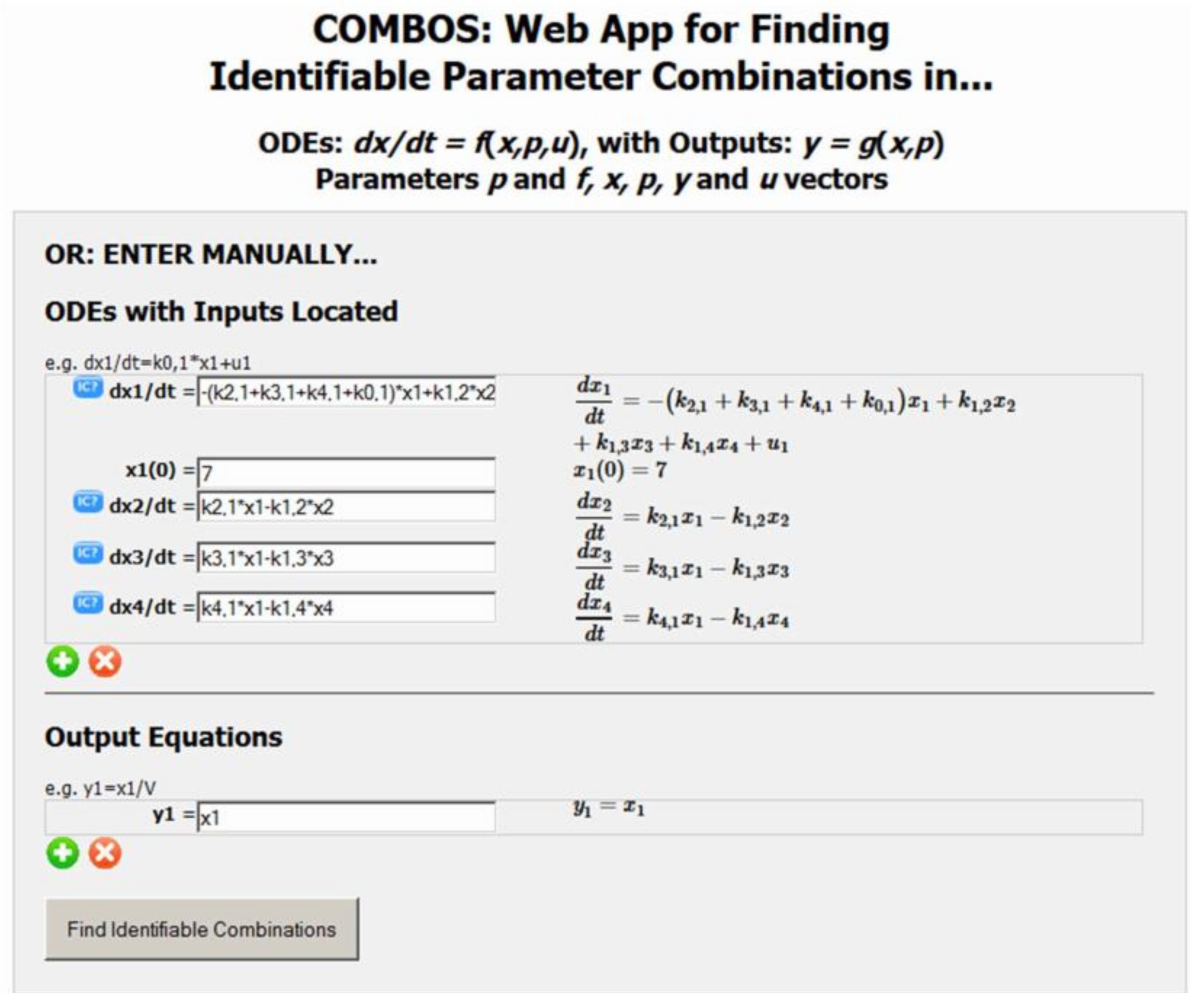

Fig. 1. Entering model parameters in COMBOS to study model identifiability [9]
Rys. 1. Wprowadzanie parametrów modelu w programie COMBOS w celu badania identyfikowalności modelu [9]

5. Identifiability testing of aggregate model parameters tractor baler-wrapper

The complex dynamic system of the tractor - single-axis agricultural machine (baler-wrapper) was tested. Tractor model dynamics - a single-axis agricultural machine correctly describes the bicycle model [9]. The machine model described in the form of ordinary differential equation (ODE) differential equations was introduced into the calculation program. In this case was used the program STRIKE-GOLDD which is a MATLAB toolbox that analyses the local structural identifiability observability and invertibility of (possibly nonlinear) dynamic models of ordinary differential equations (ODEs). It follows a differential geometry approach recasting the identifiability problem as an observability problem. Gröbner bases of model attributes are computed to find the identifiable parameters and parameter combinations of the model [8]. The identifiability is determined by calculating the rank of a generalized observability-identifiability matrix which is built using Lie derivatives.

STRIKE-GOLDD reads models stored as MATLAB MAT-files (.mat). The model states outputs and parameters as well as its dynamic equations must be defined as vectors of symbolic variables whose names must follow a specific convention. The process of defining a suitable model consists of the following stages:

- all the parameters states and any other entities (known constants) appearing in the model must be defined as symbolic variables: syms,
- define the state variables by creating a column vector named x,
- define the vector of output variables which must be named h,
- the vector of unknown parameters must be called p,
- the dynamic equations must also be entered as a column vector called f,
- the vector of initial conditions must be called ics,
- define another vector known_ics to specify which initial conditions are known. It must have the same length as the state vector x and its entries should be either 1 or 0 depending on whether the corresponding initial condition is known or unknown
- finally save all the variables in a MAT-file.

Tractor model dynamics - a single-axis agricultural machine correctly describes the bicycle model [4, 5, 6]. An example of such a machine is the aggregate tractor baler-wrapper. The machine model described in the form of ordinary differential equation (ODE) was introduced into the calculation program.

The machine model entered into the calculation program has the form [11]:

Dynamic variables

- v_tc - tractor CG lateral velocity [ms^{-1}],
- gam_t - tractor yaw rate [rads^{-1}],
- gam_i - implement yaw rate [rads^{-1}],
- alp_tf - tractor front tire slip angle [rad],
- alp_tr - tractor rear tire slip angle [rad],
- alp_ir - implement tire slip angle [rad],
- x_tc - tractor CG trajectory in world [m],
- y_tc - tractor CG trajectory in world [m],
- psi_t - tractor heading angle in world [rad],
- psi_i - implement heading angle in world [rad].

Source: own study / Źródło: opracowanie własne

Fig. 2. Tractor baler-wrapper set during field tests

Rys. 2. Agregat prasoowijarka-ciągnik podczas badań polowych

- Dynamic parameters
- C_atf - tractor front tire cornering stiffness [Nrad^{-1}],
- C_atr - tractor rear tire cornering stiffness [Nrad^{-1}],
- C_air - implement tire cornering stiffness [Nrad^{-1}],
- sig_tf - tractor front tire relaxation length [m],
- sig_tr - tractor rear tire relaxation length [m],
- sig_ir - implement tire relaxation length [m],

Constant (fixed) parameters

- m_t - tractor mass [kg],
- m_i - implement mass [kg],
- I_tz - tractor yaw moment of inertia [kg·m^2],
- I_iz - implement yaw moment of inertia [kg·m^2],
- u_tc - tractor longitudinal velocity[ms^{-1}],
- a - distance between front axle and CG of tractor [m],
- b - distance between rear axle and CG of tractor [m],
- c - distance between tractor CG and hitch point [m],
- d - distance between hitch point and implement CG [m],
- e - distance between implement CG and implement axle [m],
- del -ramp input to front wheel angle [rad].

The data is entered into the calculation program by specifying the required model parameters in the calculation procedure.

The following are examples of the parameters of the aggregate model tractor baler-wrapper (Fig. 2). The most complex is to provide the form of ordinary differential equation (ODE) describing the machine model (dynamic equations).

```
% states:
x=[v_tc;gam_t;gam_i;alp_tf;alp_tr;alp_ir;x_tc;y_tc;psi_t;psi_i]

% parameters :
p = [v_tc;gam_t;gam_i;alp_tf;alp_tr;alp_ir;x_tc;y_tc;psi_t;psi_i;C_atf;C_atr;C_air;sig_tf;sig_tr;sig_ir];

% outputs:
h=[x_tc;y_tc;gam_t;psi_t]

% one input:
u = [deld];

% dynamic equations:
f(1)= - gam_t*u_tc - (C_atf*alp_tf*(I_iz*m_i*c^2 + I_iz*a*m_i*c + I_tz*m_i*d^2 + I_iz*I_tz))/(I_iz*m_i*m_t*c^2 +
I_tz*m_i*m_t*d^2 + I_iz*I_tz*m_i + I_iz*I_tz*m_t) - (C_atr*alp_tr*(I_iz*m_i*c^2 - I_iz*b*m_i*c + I_tz*m_i*d^2 +
I_iz*I_tz))/(I_iz*m_i*m_t*c^2 + I_tz*m_i*m_t*d^2 + I_iz*I_tz*m_i + I_iz*I_tz*m_t) - (C_air*I_tz*alp_ir*(I_iz -
d*e*m_i))/(I_iz*m_i*m_t*c^2 + I_tz*m_i*m_t*d^2 + I_iz*I_tz*m_i + I_iz*I_tz*m_t)

f(2)= (C_atr*alp_tr*(b*m_i*m_t*d^2 + I_iz*b*m_i + I_iz*b*m_t - I_iz*c*m_i))/(I_iz*m_i*m_t*c^2 + I_tz*m_i*m_t*d^2
+ I_iz*I_tz*m_i + I_iz*I_tz*m_t) - (C_atf*alp_tf*(a*m_i*m_t*d^2 + I_iz*a*m_i + I_iz*a*m_t +
I_iz*c*m_i))/(I_iz*m_i*m_t*c^2 + I_tz*m_i*m_t*d^2 + I_iz*I_tz*m_i + I_iz*I_tz*m_t) + (C_air*alp_ir*c*m_t*(I_iz -
d*e*m_i))/(I_iz*m_i*m_t*c^2 + I_tz*m_i*m_t*d^2 + I_iz*I_tz*m_i + I_iz*I_tz*m_t)

f(3)= (C_air*alp_ir*(e*m_i*m_t*c^2 + I_tz*d*m_t + I_tz*e*m_i + I_tz*e*m_t))/(I_iz*m_i*m_t*c^2 + I_tz*m_i*m_t*d^2
+ I_iz*I_tz*m_i + I_iz*I_tz*m_t) - (C_atf*alp_tf*d*m_i*(I_tz - a*c*m_t))/(I_iz*m_i*m_t*c^2 + I_tz*m_i*m_t*d^2 +
I_iz*I_tz*m_i + I_iz*I_tz*m_t) - (C_atr*alp_tr*d*m_i*(I_tz + b*c*m_t))/(I_iz*m_i*m_t*c^2 + I_tz*m_i*m_t*d^2 +
I_iz*I_tz*m_i + I_iz*I_tz*m_t)

f(4)= v_tc/sig_tf - (alp_tf*u_tc)/sig_tf - (deld*u_tc)/sig_tf + (a*gam_t)/sig_tf

f(5)= v_tc/sig_tr - (alp_tr*u_tc)/sig_tr - (b*gam_t)/sig_tr

f(6)= v_tc/sig_ir - (alp_ir*u_tc)/sig_ir - (psi_i*u_tc)/sig_ir + (psi_t*u_tc)/sig_ir - (gam_i*(d + e))/sig_ir - (c*gam_t)/sig_ir

f(7)= u_tc*cos(psi_t) - v_tc*sin(psi_t)

f(8)= v_tc*cos(psi_t) + u_tc*sin(psi_t)

f(9)= gam_t

f(10)= gam_i
```

```
% initial conditions:
ics=[0 0 0 0 0 0 0 0 0 0];

% which initial conditions are known:
known_ics=[0 0 0 0 0 0 0 0 0 0];
```

After starting the calculation program we receive information about which parameters are identifiable. The result has the form.

6. Results summary

The model is structurally unidentifiable.
These parameters are identifiable:
matrix([[C_air C_atf C_atr sig_ir sig_tf sig_tr]]).
These parameters are unidentifiable:
matrix([[alp_ir alp_tf alp_tr gam_i gam_t psi_i psi_t v_tc x_tc y_tc]]).
These states are unobservable (and their initial conditions if considered unknown are unidentifiable):
matrix([[alp_ir alp_tf alp_tr gam_i psi_i v_tc]]).
These states are directly measured:
matrix([[gam_t] [x_tc] [y_tc] [psi_t]]).
These inputs are known:
deld.

Calculations show that only parameters are identifiable while variables are unidentifiable. Therefore dynamic variables must be determined during the measurement.

7. Conclusions

Investigating the identifiability of system model parameters requires the use of complex mathematical tools. For this purpose symbolic notation and differential geometry methods are used. The available software for testing the identifiability of system model parameters in the form of a network application and toolboxes in the Matlab environment was presented. In order to perform calculations it is required to provide system differential equations (ODEs) state variables and system inputs and outputs.

An example of dynamic system model identification aggregate tractor uniaxial machine was presented. The presented methods of assessing the identifiability of system parameters can be useful during parameter estimation. They allow you to assess whether it is possible to uniquely obtain model parameter values from the given data set.

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