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RELATIONSHIP BETWEEN PLOUGH BODY SHAPE AND ITS SPECIFIC RESISTANCE

Summary

The specific resistance of the plough body (a quotient of its working resistance and furrow cross section) depends on three factors: soil conditions, working speed and plough body shape. The last dependence has not been previously defined analytically. A novel method is being presented in this publication, which allows us to define and calculate the parameters of a plough body influencing its specific resistance, as well as the relationship between these parameters and specific resistance of different plough bodies. The proposed equation includes also soil conditions and working speed. The calculated parameters are based on the dimensions taken from drawing of a plough body working surface through the soil. The knowledge of such a relationship should be very useful in the design of new plough bodies and improvement of existing ones. The dependence between defined parameters and specific resistance has been calculated with statistical methods on the basis of field- and laboratory investigations of plough bodies with different working surface shapes. The scope of this investigation covers the cylindroidal, semihelicoidal and helicoidal plough bodies, their working speeds in the range of 0.81 – 2.5 m/s, and soil compactness measured with a cone penetrometer in the range of 1.35 – 1.93 MPa (in field investigations). The introduced relationship has been verified for another 10 ploughs during field investigations, conducted in different soil conditions, at different working speeds.

ZALEŻNOŚĆ POMIĘDZY KSZTAŁTEM POWIERZCHNI ROBOCZEJ KORPUSU PŁUŻNEGO A JEGO OPOREM JEDNOSTKOWYM

Streszczenie

Opór roboczy korpusu płużnego zależy od trzech parametrów: warunków glebowych, prędkości roboczej i kształtu powierzchni roboczej korpusu płużnego. Prezentowana zależność nie była dotychczas zdefiniowana analitycznie. Metoda prezentowana w tej publikacji pozwala określić i obliczyć parametry kształtu korpusu płużnego wpływające na opór jednostkowy, pokazuje także wpływ tych parametrów na opór dla różnych korpusów płużnych. Zaproponowane równie uwzględnia również warunki glebowe i prędkość roboczą. Wskaźniki charakteryzujące kształt korpusu płużnego wyznaczono z profilogramów. Prezentowana metoda i zależność może być przydatna przy projektowaniu nowych korpusów płużnych jak również przy poprawieniu kształtu już istniejących korpusów. Równanie charakteryzujące opór jednostkowy korpusów płużnych, uwzględniające parametry kształtu korpusów płużnych i parametry glebowe, zostało uzyskane przy zastosowaniu metod statystycznych w oparciu o badania polowe i laboratoryjne korpusów płużnych o różnym kształcie powierzchni roboczej. Zakres badań obejmował korpusy płużne cylindroidalne, półśrubowe i śrubowe, w badaniach stosowano prędkości robocze w zakresie 0,81 – 2,5 m/s przy zwięzłości gleby w przedziale 1,35 – 1,93 MPa mierzonej penetrometrem stożkowym (w badaniach polowych). Do weryfikacji zależności wykorzystano badania polowe 10 korpusów płużnych prowadzone w zróżnicowanych warunkach glebowych przy różnych prędkościach roboczych.

1. Introduction

The comparative estimation of specific resistance for different plough bodies allows for selection of the best parameters of their working surface during their design, before their experimental investigations. Bernacki [1] proposed a relationship between the plough body resistance and the angle of its mouldboard setting. Sommerburg [5] searched for a relationship between selected parameters of plough body and its resistance. His equation was discussed and improved by Ganzuch et al. [2] as well as by Kubisch [3]. Richey et al. [4] describe the surface of plough body by the bicubic equation and searched for its connections with the soil-tool mechanics model. However, all these elaborations are not satisfactory for forecasting the specific resistance of different plough bodies, on the base of theirs project drawings.

2. Methodology

Method of plough body parameters calculation

Projected plough body surface is defined by the set of dimensions, given in the table on its drawing, as the coordinates of points in vertical cross-sections, perpendicular to the edge of share (Fig. 1).

Using these coordinates, the plough body surface has been described by the following equation:

$$x1 = f\left[\left(z^3+z^2+z\right)\left(1+y1+y1^2+y1^3\right) + \left(y1^3+y1^2+y1\right)\right] \qquad (1)$$

where: *x1*, *y1*, *z* - are the coordinates of plough body surface (perpendicular and parallel to the edge of share), taken from the table on the drawing.

The {do you mean the “above”} equation has been developed to the form:

$$\begin{aligned} x1 = {} & a_1 z^3 + a_2 z^2 + a_3 z + a_4 z^3 y1 + a_5 z^2 y1 + a_6 z\, y1 + a_7 z^3 y1^2 + \\ & + a_8 z^2 y1^2 + a_9 z\, y1^2 + a_{10} z^3 y1^3 + a_{11} z^2 y1^3 + a_{12} z\, y1^3 + \\ & + a_{13} y1^3 + a_{14} y1^2 + a_{15} y1 \end{aligned} \qquad (2)$$

In this equation $a_1 \div a_{15}$ are multiple regression coefficients, and only the ones with statistically significant *t*-test values were considered during calculation.

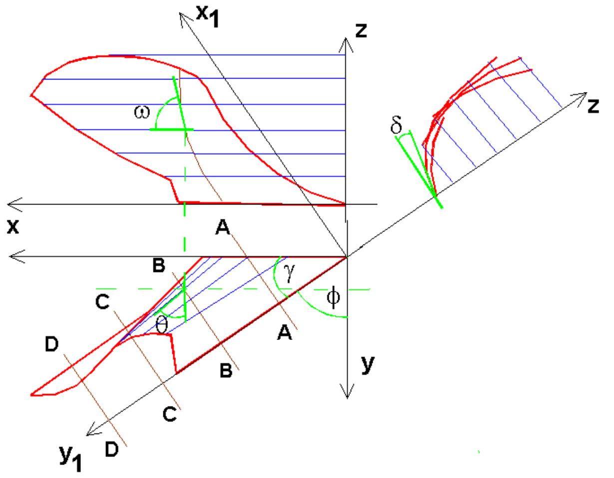

Fig. 1. Main elements of a typical plough body drawing: x, y, z – coordinates system parallel and perpendicular to the plough movement, x1, y1, z – coordinates system perpendicular and parallel to the share edge, γ, φ - share edge angle and its geometrical complement, ω, θ, - angles, calculated with formulas (5) and (9), δ - share cutting angle, A-A, B-B, C-C, D-D – cross sections of the mouldboard and their shapes, (only selected horizontal contour lines are shown on the drawing)

The following function was formulated, in the system of coordinates: parallel and perpendicular to the plough movement, using Euler's transformation for rotation of coordinates:

$$f1(z, x, y) = 0 \tag{3}$$

where:

$$x = x1\cos\varphi + y1\sin\varphi; \qquad y = -x1\sin\varphi + y1\cos\varphi \tag{4}$$

The angle φ is between the edge of the plough body share and perpendicular to its movement.

The angles ω were calculated in vertical planes, parallel to the plough body movement, for all points of the network on the regression surface, as between the tangent to the surface and horizontal. The angles were calculated as derivatives of the function, given in equation (3):

$$\omega = arctan\left(\frac{dz}{dx}\right) = -\frac{\dfrac{\partial f1}{\partial x}}{\dfrac{\partial f1}{\partial z}} \tag{5}$$

The partial derivatives were calculated as follows:

$$\frac{\partial f1}{\partial x} = \begin{pmatrix} \left(a_4 + 2a_7 W + 3a_{10} W^2\right) z^3 + \left(a_5 + 2a_8 W + 3a_{11} W^2\right) z^2 + \\ + \left(a_6 + 2a_9 W + 3a_{12} W^2\right) z + \left(a_{15} + 2a_{14} W + 3a_{13} W^2\right) \end{pmatrix} \sin\varphi - \cos\varphi \tag{6}$$

$$\frac{\partial f1}{\partial z} = 3\left(a_1 + a_4 W + a_7 W^2 + a_{10} W^3\right) z^2 + 2\left(a_2 + a_5 W + a_8 W^2 + a_{11} W^3\right) z + \left(a_3 + a_6 W + a_9 W^2 + a_{12} W^3\right) \tag{7}$$

Coefficient W was introduced:

$$W = x \sin\varphi + y \cos\varphi \tag{8}$$

The angles θ in horizontal planes were calculated in the same way, as between the tangent to the surface and perpendicular to the plough body movement. These angles were calculated as derivatives:

$$\theta = arctan\left(\frac{dx}{dy}\right) = -\frac{\dfrac{\partial f2}{\partial y}}{\dfrac{\partial f2}{\partial x}} \tag{9}$$

with adequate form of function $f2$ and adequate formulas and coefficients for partial derivatives.

The set of mean angles was selected on the basis of these calculated angles, as parameters of plough body, connected to their specific resistance (table 1). The mean cutting angle Δ of share in plane perpendicular to its edge was also introduced as one of the parameters. All angles were taken in radians.

For calculation of these angles was carried out by a program written specifically for the task in Pascal language.

Table 1. Selected parameters of plough body, presented in the analysis of the relationship between plough body shape and its specific resistance

Parameter of the shape of working surface	Description
$\boldsymbol{\Omega s}$	Mean value of the angles ω in vertical plane, for the whole working surface
$\boldsymbol{\Omega z}$	Mean value of the angles ω in vertical plane, at the highest contour line
$\boldsymbol{\Omega n}$	Mean value of the angles ω in vertical plane, at the contour line for rated depth of ploughing of the examined plough body
$\boldsymbol{\Theta s}$	Mean value of the angles θ in horizontal plane, for the whole working surface
$\boldsymbol{\Theta n}$	Mean value of the angles θ in horizontal plane, at the contour line for rated depth of ploughing of the examined plough body
$\boldsymbol{\Theta r}$	Difference of the maximum and minimum values of the angles θ in horizontal plane
$\boldsymbol{\Delta}$	Cutting angle of the share, in the plane perpendicular to its edge

3. Results and discussion

Specific resistance of plough body as related to its shape

The measurements of specific resistance for 7 plough body shapes were conducted in order to determine the relationship between plough body shape and its specific resistance (3 shapes in a soil bin and 4 shapes in field investigations). The soil cone index "***p***" (at the full depth of ploughing), water content in the soil "***u***" and working speed "***v***" were determined during all measurements. The seven characteristic angles (see table 1) were also calculated for each plough body. The angles were combined by multiplication with ***p, u, v*** as well as with $\boldsymbol{p \times u}$, $\boldsymbol{p \times v}$, $\boldsymbol{u \times v}$ and $\boldsymbol{p \times u \times v}$. The matrix of 7 angles × 7 working conditions was determined as a result, and it gave for all plough bodies and theirs working conditions a set of 2597 values, with different specific resistance and different characteristics of working surface, related to the soil conditions.

The following equation was found, using a stepwise regression, additionally corrected on the logical base:

$$\begin{aligned} \boldsymbol{k} = & \left[\begin{array}{l} \boldsymbol{p}\left(16{,}956\,\Delta\right) + \boldsymbol{u}\left(-177{,}41\,\Omega z + 297{,}345\,\Omega n + 166{,}650\,\Theta r\right) \\ + \boldsymbol{p}\,\boldsymbol{u}\left(-12{,}862\,\Omega n - 15{,}202\,\Theta s\right) \end{array} \right] \\ & + \left[\begin{array}{l} \boldsymbol{p}\left(8{,}666\,\Omega s - 2{,}680\,\Omega z - 16{,}327\,\Delta\right) + \boldsymbol{u}\left(223{,}292\,\Theta s - 337{,}897\,\Theta n\right) \\ + \boldsymbol{p}\,\boldsymbol{u}\left(-3{,}951\,\Omega n + 78{,}619\,\Delta\right) + 16{,}662\,\Theta n \end{array} \right] \boldsymbol{v}^2 \end{aligned} \tag{10}$$

where:
k – specific resistance of the plough body [kN/m^2],
p – cone index [kN/m^2],
u – water content in the soil [kg of water/kg of dry mass],
v – working speed [m/s], all angles given in table 1 [radians].

The results of large field investigations of 10 different ploughs shapes were taken, in order to verify the obtained equation. These investigations were conducted during three years, of several soil types and working conditions. The drawings of plough body working surfaces were in the know before the field investigations. Mean values of measured specific resistance were taken into consideration for each plough body, soil condition and working speed. For the same parameters specific resistances were calculated using formula (10). Both, measured and calculated results are compared in Fig. 2.

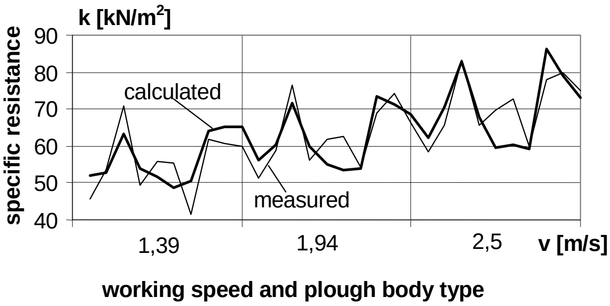

Fig. 2. Calculated and measured values of specific resistance of different plough bodies: bending points of lines refer to the different plough bodies

The analysis of differences between measured and calculated values showed that there is not a statistically significant difference between both sets of values at the 95.0% confidence level. The absolute error value is 4.55 kN/m^2, which is 7.2% of the mean value. The correlation coefficient of the linear regression between calculated and measured values was R = 0.848. The statistical analysis indicate, that the proposed relationship between the plough body shape and its specific resistance has been satisfactorily confirmed by the field investigations.

4. Conclusions

a) Plough body shape may be mathematically described by a bicubic equation and its working surface is defined with seven angles, introduced in this publication.

b) The elaborated equation, based on the introduced angles as well as on the soil conditions and working speed, allows for comparative forecasting of specific resistance of different plough bodies, using the data from the plough drawing design.

c) The results of this elaborated equation were analysed by statistical methods and may be considered valid within the range of soil parameters and working speed considered in this paper.

5. References

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