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A MATHEMATICAL MODEL OF OSCILLATIONS OF TRAILED AGRICULTURAL MACHINES MOVING ALONG THE IRREGULARITIES OF SOIL

Summary

Oscillatory movements of trailed agricultural machines have been studied theoretically and analysed in the process of their movement along the surface irregularities of soil. On the basis of differential equations of the movement of a mechanical system in a longitudinal vertical plane (using the Lagrange equations of the 2nd kind) a mathematical model of the oscillatory movement of a trailed agricultural machine aggregate due to the soil irregularities has been developed.

1. Introduction

An article was devoted to the study of oscillatory movements of self-propelled agricultural machines and machine aggregates in this magazine in 2007 [1]. The quality of the technological process performed by many agricultural machines depends to a certain degree on the stability of their movement in a vertical plane when they are running along the surface irregularities of soil [2, 3]. However, in contrast to the self-propelled machines, the movement of the towed machines and machine aggregates along the surface irregularities of soil has a series of differential features, and an analytical description of this process will be different. The operating tools of a self-propelled machine aggregate are hanged onto the tractor undercarriage, and the operating tools, together with the wheels of the tractor, form an integrated unit, and their oscillatory movements take place simultaneously. In a towed machine aggregate this process represents a more complicated mechanical system because only a part of the machine has a support on the tractor, the other part independently copying the surface irregularities of soil. On the basis of this assumption, the authors thought it urgent to consider this issue in greater detail.

2. Materials and methods

The aim of this investigation is to establish the degree of the impact of oscillatory movements of agricultural machine aggregates on the quality indices of their work. In order to reach the advanced aim, theoretical studies were applied based on differential equations of the movement of a mechanical system using the Lagrange equations of the 2nd kind.

3. Results and discussion.

Let us build an equivalent calculation scheme of the oscillatory movements of a towed agricultural machine aggregate (Fig. 1.). It will be a mechanical system with one degree of freedom.

Letter designations: *Oxyz* – a fixed system of coordinates (plane *xOz* is a vertical plane which is perpendicular to the plane of the field), z – vertical displacement of the centre of mass of the aggregate; *C*- centre of masses of the system; l_1 – the distance from the frontal point of the suspension to the centre of masses of the system; l_2 – the distance of the rear point of the suspension to the centre of masses of the system; l – the distance from the frontal point to the rear point of the suspension of the system, c – suspension stiffness of the machine aggregate; h – the height of the irregularity of the bearing surface of the soil under the wheels, μ – the resistance coefficient of the suspension and the tyres of the machine aggregate.

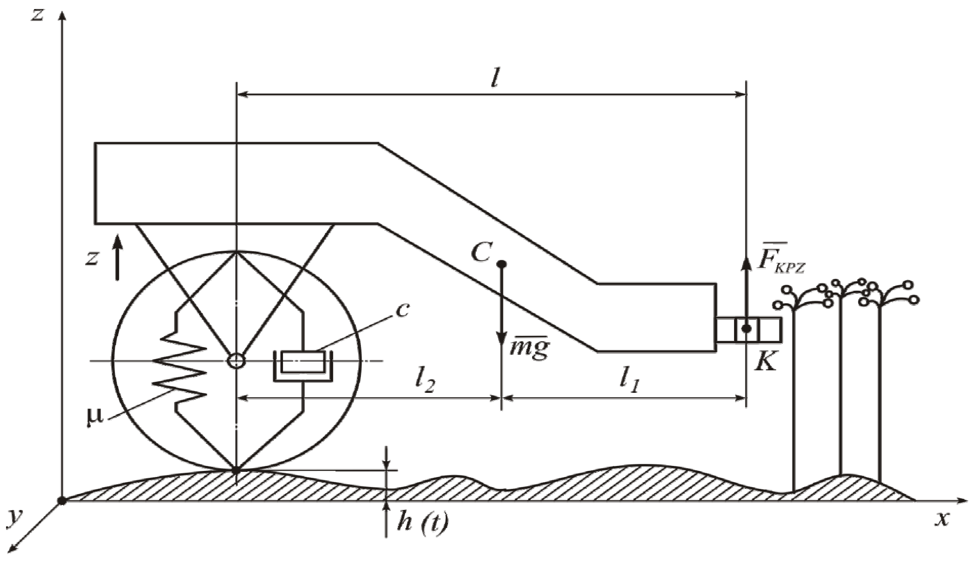

Fig. 1. An equivalent scheme of a machine aggregate reduced to an oscillatory mechanical system with one degree of freedom

As a generalised coordinate we take the vertical movement z of a sprung mass over the rear running wheels (there are no front wheels). We will start counting off the generalised coordinate z from the position of static equilibrium of the system. Then the movement of this mechanical system will also be described in the form of a Lagrange equation of the 2[nd] kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}}\right)-\frac{\partial T}{\partial z}=Q_z, \qquad (1)$$

where $T=\frac{1}{2}m\dot{z}^2$,

$$\Pi=\frac{1}{2}c(z-h)^2,$$

$$h=h(t),$$

$$\Phi=\frac{1}{2}\mu\left(\dot{z}-\dot{h}\right)^2,$$

$m=\frac{M\,l_1}{l}$ – the mass of a part of the machine aggregate which performs the vertical oscillatory movements.

Let us determine the generalised force for this instance of the movement of the machine aggregate. It will be equal to:

$$Q_z=Q_z^{(\Pi)}+Q_z^{(\Phi)}+Q_z^{(B)}, \qquad (2)$$

where

$$Q_z^{(\Pi)}=-\frac{\partial \Pi}{\partial z}=-c(z-h),$$

$$Q_z^{(\Phi)}=-\frac{\partial \Phi}{\partial \dot{z}}=-\mu\left(\dot{z}-\dot{h}\right),$$

$$Q_z^{(B)}=0,$$

$$Q_z=-c(z-h)-\mu\left(\dot{z}-\dot{h}\right).$$

We perform the necessary transformations for (1).

We have:

$$\frac{\partial T}{\partial \dot{z}}=m\dot{z},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}}\right)=m\ddot{z},$$

$$\frac{\partial T}{\partial z}=0.$$

By substituting (2) into (1) and using the performed trans $m\ddot{z}=-c(z-h)-\mu\left(\dot{z}-\dot{h}\right)$,

or:

$$\ddot{z}=-\frac{c}{m}(z-h)-\frac{\mu}{m}\left(\dot{z}-\dot{h}\right),$$

or:

$$\ddot{z}+\frac{\mu}{m}\dot{z}+\frac{c}{m}z=\frac{ch}{m}+\frac{\mu}{m}\dot{h}, \qquad (3)$$

Let

$$\frac{c}{m}=k^2,$$

$$\frac{\mu}{2m}=n.$$

Then the differential equation (3) will assume the following appearance:

$$\ddot{z}+2n\dot{z}+k^2z=k^2h(t)+2n\dot{h}(t). \qquad (4)$$

It is known that the common solution of the differential equation (4) is equal to:

$$z=z_1+z_2, \qquad (5)$$

where z_1 – the common solution of the homogeneous differential equation

$$\ddot{z}+2n\dot{z}+k^2z=0, \qquad (6)$$

but z_2 – a partial solution of the nonhomogeneous differential equation depending on the kind of the right side.

According to the theory of differential equations, the common solution of the differential equation (6) can have one of the following appearances:

1. If there is low resistance $(n<k)$:

$$z_1(t)=e^{-nt}(C_1\cos k_1t+C_2\sin k_1t),\ k_1=\sqrt{k^2-n^2}, \qquad (7)$$

or

$$z_1(t)=ae^{-nt}\sin(k_1t+\beta).$$

2. If there is high resistance $(n>k)$:

$$z_1(t)=e^{-nt}\left(C_1e^{k_2t}+C_2e^{-k_2t}\right),\ k_2=\sqrt{n^2-k^2}. \qquad (8)$$

3. If there is critical resistance $(n=k)$:

$$z_1(t)=e^{-nt}(C_1+C_2t). \qquad (9)$$

In expressions (7-9) C_1 and C_2 – arbitrary constants, which are determined from the initial conditions.

Instances 2 and 3 are fading nonoscillatory movements. Instance 1 – free fading oscillations with amplitude ae^{-nt} frequency k_1.

Structure $z_2(t)$ of a partial solution of the differential equation (1) depends on the surface irregularities of soil, i.e. on the kind of function $h(t)$.

Let the irregularities of soil be described with a certain approximation by the following harmonic function:

$$h(t)=h_o\sin\left(\frac{Vt}{L}\right), \qquad (10)$$

where $h(t)$ – the height of the irregularities of soil; L – the length of the irregularities of soil; V –the forward speed of the movement of the machine aggregate; (h_o, V, L) – the parameters the values of which are assigned.

Let us define further

$$\frac{V}{L}=k_3.$$

Then expression (10) will assume such an appearance:

$$h(t) = h_o \sin k_3 t \,. \qquad (11)$$

By substituting expression (11) into the differential equation (4) we obtain:

$$\ddot{z} + 2n\dot{z} + k^2 z = k^2 h_o \sin k_3 t + + 2nh_o k_3 \cos k_3 t \,. \qquad (12)$$

Then the partial solution z_2 of equation (12) should be searched for in the following way:

$$z_2 = A \sin k_3 t + B \cos k_3 t \,, \qquad (13)$$

where A and B – unknown coefficients.

Let us determine coefficients A and B by the method of uncertain coefficients. For this we find the necessary derivatives:

$$\dot{z}_2 = Ak_3 \cos k_3 t - Bk_3 \sin k_3 t \,, \qquad (14)$$

$$\ddot{z}_2 = -Ak_3^2 \sin k_3 t - Bk_3^2 \cos k_3 t \,. \qquad (15)$$

By substituting expressions (14) and (15) into (12) we obtain:

$$\begin{aligned} & -Ak_3^2 \sin k_3 t - Bk_3^2 \cos k_3 t + \\ & + 2nAk_3 \cos k_3 t - \\ & - 2nBk_3 \sin k_3 t + k^2 A \sin k_3 t + \\ & + k^2 B \cos k_3 t = k^2 h_o \sin k_3 t + \\ & + 2nh_o k_3 \cos k_3 t. \end{aligned} \qquad (16)$$

By equating the coefficients at equal functions in the left and the right side of expression (16) we obtain the following system of algebraic equations in relation to the unknowns A and B:

$$\left.\begin{aligned} & -Ak_3^2 - 2nBk_3 + k^2 A = k^2 h_o \,, \\ & -Bk_3^2 + 2nAk_3 + k^2 B = 2nh_o k_3 \,. \end{aligned}\right\} \qquad (17)$$

We apply the Cramer method to solve the system of equations (17), and therefore we rewrite this system in the following way:

$$\left.\begin{aligned} & \left(k^2 - k_3^2\right)A - 2nk_3 B = k^2 h_o \,, \\ & 2nk_3 A + \left(k^2 - k_3^2\right)B = 2nh_o k_3 \,. \end{aligned}\right\} \qquad (18)$$

Let us calculate the necessary determiners:

$$\Delta = \begin{vmatrix} k^2 - k_3^2 & -2nk_3 \\ 2nk_3 & k^2 - k_3^2 \end{vmatrix} = \left(k^2 - k_3^2\right)^2 + 4n^2 k_3^2 \,,$$

$$\Delta_A = \begin{vmatrix} k^2 h_o & -2nk_3 \\ 2nh_o k_3 & k^2 - k_3^2 \end{vmatrix} = h_o \left[k^2 \left(k^2 - k_3^2\right) + 4n^2 k]_3^2 \right],$$

$$\Delta_B = \begin{vmatrix} k^2 - k_3^2 & k^2 h_o \\ 2nk_3 & 2nh_o k_3 \end{vmatrix} = -2nk_3^3 h_o \,.$$

Then:

$$A = \frac{\Delta_A}{\Delta} = \frac{h_o \left[k^2 \left(k^2 - k_3^2\right) + 4n^2 k_3^2 \right]}{\left(k^2 - k_3^2\right)^2 + 4n^2 k_3^2} \,, \qquad (19)$$

$$B = \frac{\Delta_B}{\Delta} = -\frac{2nk_3^3 h_o}{\left(k^2 - k_3^2\right) + 4nk_3^2} \,. \qquad (20)$$

Consequently, a partial solution $z_2(t)$ follows from expression (13) where coefficients A and B are deduced from expressions (19) and (20), respectively. It is known that expression (13) can be written in the following way:

$$z_2(t) = H \sin\left(k_3 t + \beta_3\right), \qquad (21)$$

where $H = \sqrt{A^2 + B^2}$,

$$tg\beta_3 = B/A \,. \qquad (22)$$

Expression (21) describes the forced oscillations of the agricultural machine aggregate in a longitudinal vertical plane with amplitude H and frequency k_3.

Besides, the number

$$\beta_3 = arctg\, B/A \qquad (23)$$

is the initial stage of the forced oscillations of the machine aggregate.

Thus, taking into account (5), the common solution of the differential equation (12) can be written in one of the following forms:

1. If there is low resistance $(n < k)$:

$$z(t) = e^{-nt}\left[C_1 \cos k_1 t + C_2 \sin k_1 t\right] + + A \sin k_3 t + B \cos k_3 t \,,$$

or

$$z(t) = ae^{-nt} \sin\left(k_1 t + \beta\right) + H \sin\left(k_3 t + \beta_3\right) \qquad (24)$$

2. If there is high resistance $(n > k)$:

$$z(t) = e^{-nt}\left(C_1 e^{k_2 t} + C_2 e^{-k_2 t}\right) + + A \sin k_3 t + B \cos k_3 t \,, \text{ or}$$

$$z(t) = e^{-nt}\left(C_1 e^{k_2 t} + C_2 e^{-k_2 t}\right) + + H \sin\left(k_3 t + \beta_3\right) \,. \qquad (25)$$

3. If there is critical resistance $n = k$:

$$z(t) = e^{-nt}\left(C_1 + C_2 t\right) + A \sin k_3 t + B \cos k_3 t \text{ or}$$

$$z(t) = e^{-nt}\left(C_1 + C_2 t\right) + H \sin\left(k_3 t + \beta_3\right). \qquad (26)$$

The arbitrary constants C_1 and C_2 are determined from the following initial conditions:

at $t = 0$:

$$z = 0, \; \dot{z} = 0. \qquad (27)$$

If there is high or critical resistance, then the nonoscillatory movements fade rather quickly, and therefore at $t > T$, where T – a certain moment of time, one can consider that

$$z(t) \approx H \sin\left(k_3 t + \beta_3\right), \qquad (28)$$

i.e. the oscillations of the machine aggregate take place only at the expense of forced oscillations. In the case of low resistance ($n < k$) the free fading oscillations can essentially influence the oscillatory process of the towed machine aggregate.

Since in the case of low resistance ($n < k$) the entire oscillatory process of the towed machine aggregate is described by the differential equation (24), we find out the arbitrary constants C_1 and C_2 from the initial conditions (27).

By differentiation of expression (24) in time t we obtain:

$$\dot{z}(t) = -ne^{-nt}\left(C_1 \cos k_1 t + C_2 \sin k_1 t\right) + e^{-nt}\left(-k_1 C_1 \sin k_1 t + k_1 C_2 \cos k_1 t\right) + k_3 A \cos k_3 t - k_3 B \sin k_3 t. \quad (29)$$

Taking into account the initial conditions (28), we will obtain a system of algebraic equations in relation to the unknowns C_1 and C_2:

$$\left.\begin{aligned} -nC_1 + k_1 C_2 + k_3 A = 0, \\ C_1 + B = 0. \end{aligned}\right\} \quad (30)$$

From the obtained system of equations we find:

$$C_1 = -B, \quad C_2 = -\frac{nB + k_3 A}{k_1}. \quad (31)$$

By substituting the values of C_1 and C_2 from (31) into expression (24) we obtain a rule of the oscillatory movements of the towed machine aggregate in a vertical plane arising from the impact of soil irregularities:

$$z(t) = -e^{-nt}\left(B \cos k_1 t + \frac{nB + k_3 A}{k_1} \sin k_1 t\right) + A \sin k_3 t + B \cos k_3 t, \quad (32)$$

where coefficients A and B are deduced from expressions (19) and (20), respectively.

Let us write down expression (32) in the following way:

$$z(t) = -\alpha e^{-nt} \sin(k_1 t + \beta) + H \sin(k_3 t + \beta_3), \quad (33)$$

where $\alpha = \sqrt{B^2 + \frac{(nB + k_3 A)^2}{k_1^2}}$,

$$\beta = arctg\, \frac{k_1 B}{nB + k_3 A}, \quad (34)$$

H and β_3 are determined according to expressions (22) and (23), respectively.

4. Conclusions

1. During their movement the towed agricultural machine aggregates copy the surface irregularities of soil and cause oscillations of the pneumatic supporting running wheels. In response to this the operating tools deviate from the rectilinear course of the movement, which is a reason for the lower quality of the technological process of the performed work.
2. A mathematical model has been worked out that allow finding in an analytical way the conditions for the stabilisation of the movement of towed agricultural machines in a longitudinal vertical plane.

5. References

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